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Dissipative Particle Dynamics

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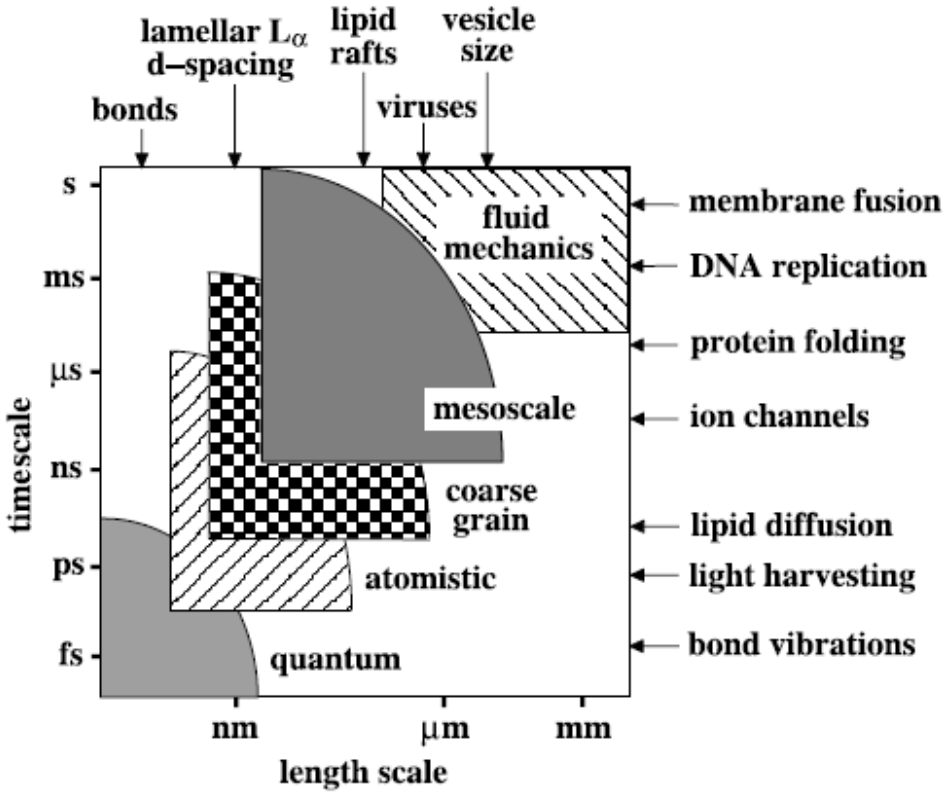
Mesoscale modelling



Source: Nielsen *et al.*, *J Phys Cond Matter*, **16**, R481–R512 (2004)

- Fills simulation gap between atomistic and macroscopic methods
- Both thermodynamics and hydrodynamics involved
  - Need atom-like and fluid-like effects
- Many complex systems act at mesoscale, e.g.
  - Surfaces and interfaces
  - Microfluidics
  - Biological membranes
  - Phase behaviour

Physical scales

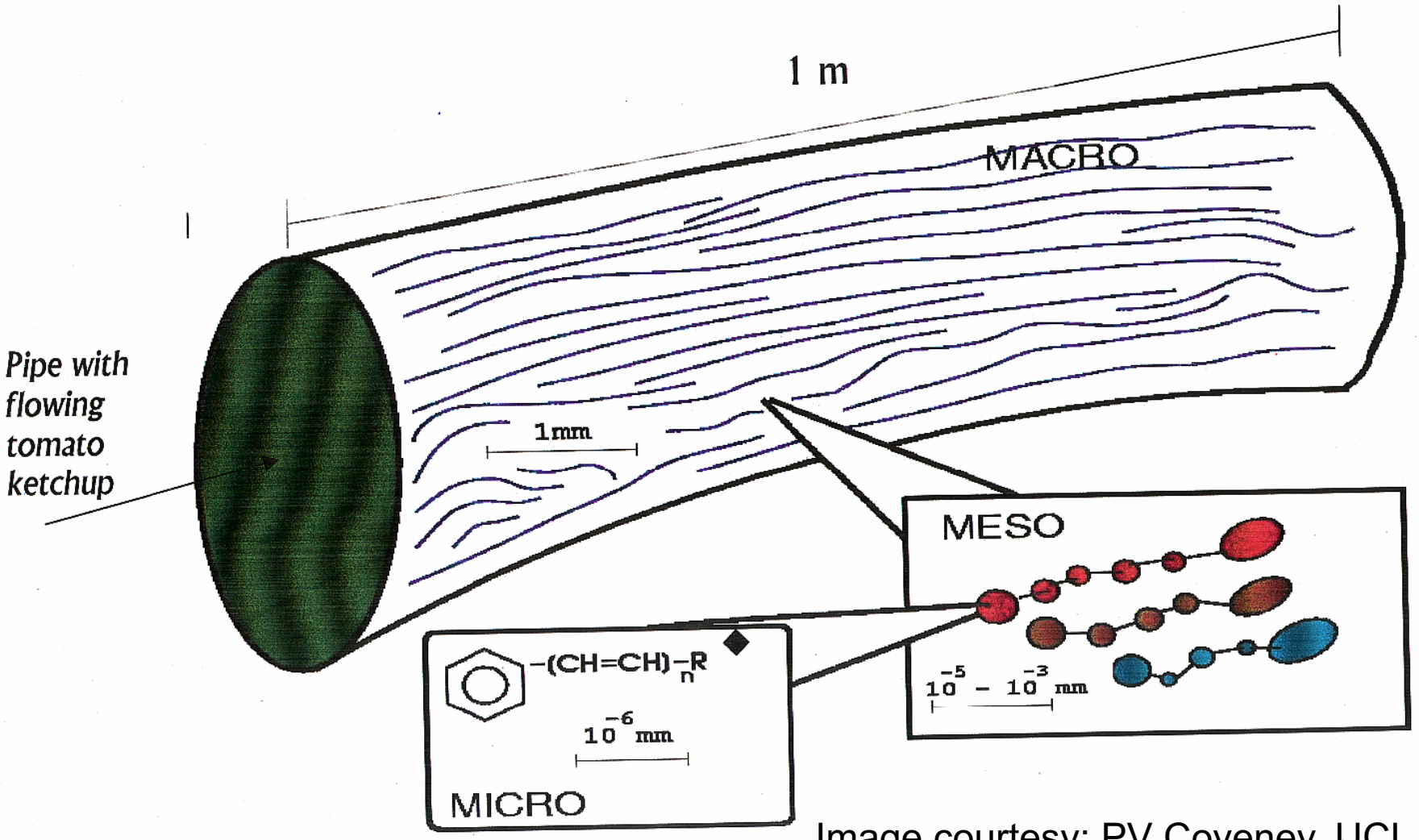


Image courtesy: PV Coveney, UCL

Mesoscale modelling techniques

Lattice Gas Cellular Automata	*LGCA*	*Hydrokinetic methods*
Multiple Component LGCA		
Lattice Boltzmann Equation	*LBE*	
Multiple Component LBE		
Dissipative Particle Dynamics	*DPD*	
Smooth particle hydrodynamics	*SPH*	
Langevin Equation / Brownian Dynamics / Stokesian Dynamics	*SD*	

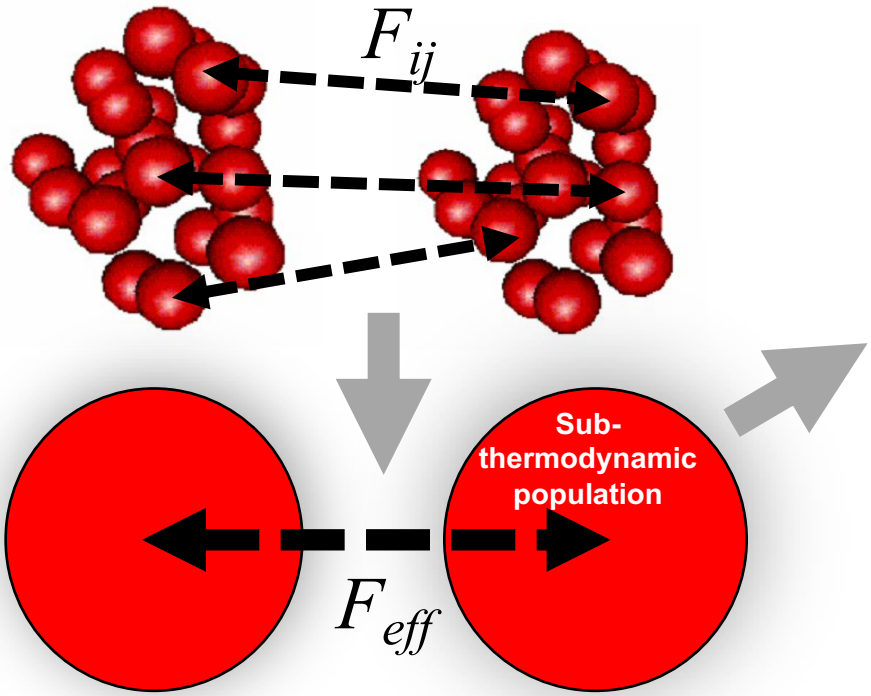
Mesoscale simulation strategies

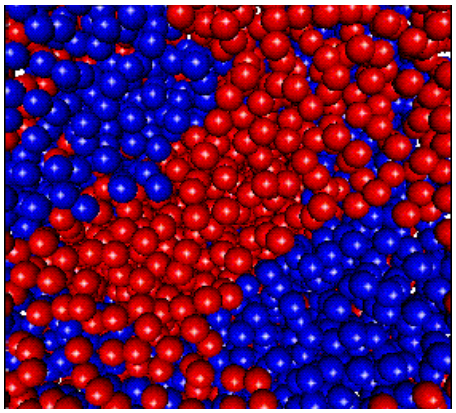
- Bottom-up: Construct methodology for coarse-graining:
  - (a) microscopic particles into sub-thermodynamic populations of mesoscopic particles, and
  - (b) their force interactions

  These result in a commensurate reduction in number of degrees of freedom

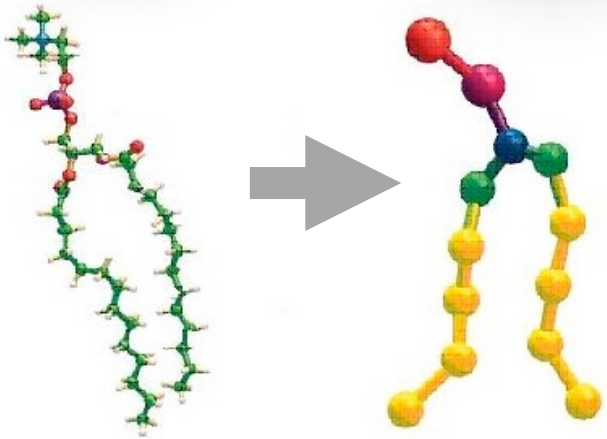
- Top-down: Reverse engineer simulation rules to obtain desired behaviour (process may, in itself, provide insight)

Mesoscale simulation strategies





In a **simple liquid**, a single atom/molecule moves in the force-field of many neighbours and interacts or **thermalizes** strongly: O(10^2) particles required to make a **sub-thermodynamic population**



Dissipative Particle Dynamics

- A brief (incomplete!) history
  - 1992: Hoogerbrugge and Koelman[1] devised DPD as off-lattice LGCA
  - 1995: Español and Warren[2] provided enhanced theoretical basis to DPD
  - 1997: Groot and Warren[3] devised systematic method to obtain DPD parameters for mesoscopic simulation
  - 1999: Flekkøy and Coveney[4] demonstrated link between MD and DPD via coarse-graining and ensemble averaging
  - 2003: Español and Revenga[5] combined DPD and SPH to give thermodynamically consistent model (SDPD)

[1] Hoogerbrugge and Koelman, *EPL* **19**, 155–160 (1992)
[2] Español and Warren, *EPL* **30**, 191–196 (1995)
[3] Groot and Warren, *J Chem Phys* **107**, 4423–4435 (1997)
[4] Flekkøy and Coveney, *Phys Rev Lett* **83**, 1775–1778 (1999)
[5] Español and Revenga, *Phys Rev E* **67**, 026705 (2003)

DPD algorithm

- Similar to classical MD
- Condensed phase system modelled by particles interacting via pairwise forces (usually soft)
- System coupled to a heat bath using pairwise stochastic and drag forces
- At equilibrium (steady state), DPD system properties calculated as ensemble averages

DPD algorithm

- DPD computational ‘particles’ (‘beads’)
  - Either populations of physical particles (coarse-graining) or ‘momentum carriers’
  - If soft, are ‘transparent’ and *can pass through each other*
  - Only discretising time, not position nor velocity
  - Newtonian dynamics: described by set of *stochastic differential equations*

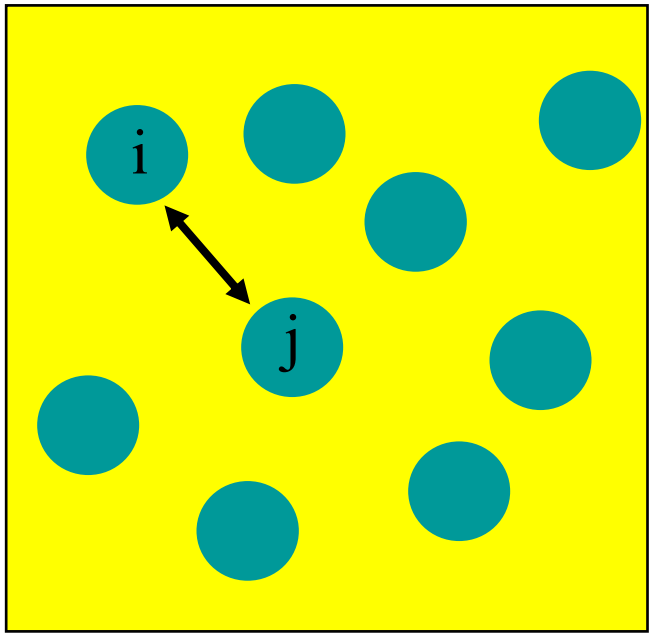
$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$$

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt}$$

DPD equations of motion

Subscript i identifies the ith DPD particle

DPD algorithm



- Pairwise forces within short distance (cut-off) calculated
- Sum of forces determines bead motion:

$$\frac{d^2\mathbf{r}_i}{dt} = \frac{1}{m_i}\left(\sum_{j\neq i}\mathbf{F}_{ij}^{C} + \sum_{j\neq i}\mathbf{F}_{ij}^{D} + \sum_{j\neq i}\mathbf{F}_{ij}^{R}\right)$$

- Forces integrated numerically over time step Δt

DPD algorithm

- Dissipative force[1]

$$\mathbf{F}_{ij}^{D} = -\gamma w^{D}(r_{ij})(\hat{\mathbf{r}}_{ij} \cdot \mathbf{v}_{ij})\hat{\mathbf{r}}_{ij}$$

where $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i$

$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$

$\hat{\mathbf{r}}_{ij} = \mathbf{r}_{ij}/r_{ij}$

  - Removes kinetic energy from centre-of-mass frame of particle pair (proportional to relative velocity)

- Random force

$$\mathbf{F}_{ij}^{R} = \sigma w^{R}(r_{ij})\theta_{ij}\hat{\mathbf{r}}_{ij}$$

defining θ_{ij} as Gaussian fluctuations, such that:

$$\langle \theta_{ij}(t) \rangle = 0, \langle \theta_{ij}(t)\theta_{i'j'}(t') \rangle = (\delta_{ii'}\delta_{jj'} + \delta_{ij'}\delta_{ji'})\delta(t - t')$$

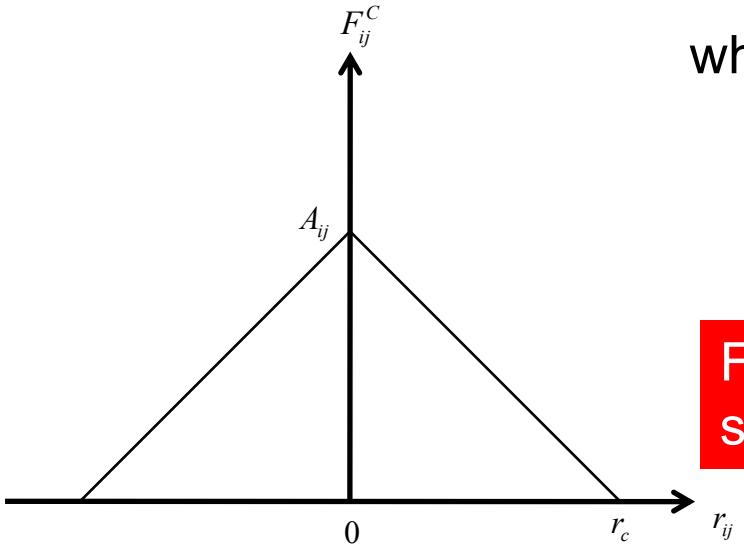
  - Drives Brownian fluctuations

[1] Groot and Warren, *J Chem Phys* **107**, 4423–4435 (1997)

DPD algorithm

- Conservative force
  - Can take any form, but usually defined as[1]:

$$\mathbf{F}_{ij}^{C} = \begin{cases} A_{ij} w^{C}(r_{ij}) \hat{\mathbf{r}}_{ij}, & r_{ij} < r_c \\ 0, & r_{ij} \geq r_c \end{cases}$$



where r_c = Cut-off radius

A_{ij} = Maximum repulsion

$w^{C}(r_{ij}) = 1 - r_{ij}/r_c$

Finite force/potential at zero separation: soft repulsive force

[1] Groot and Warren, *J Chem Phys* **107**, 4423–4435 (1997)

DPD algorithm

- Groot-Warren potential:

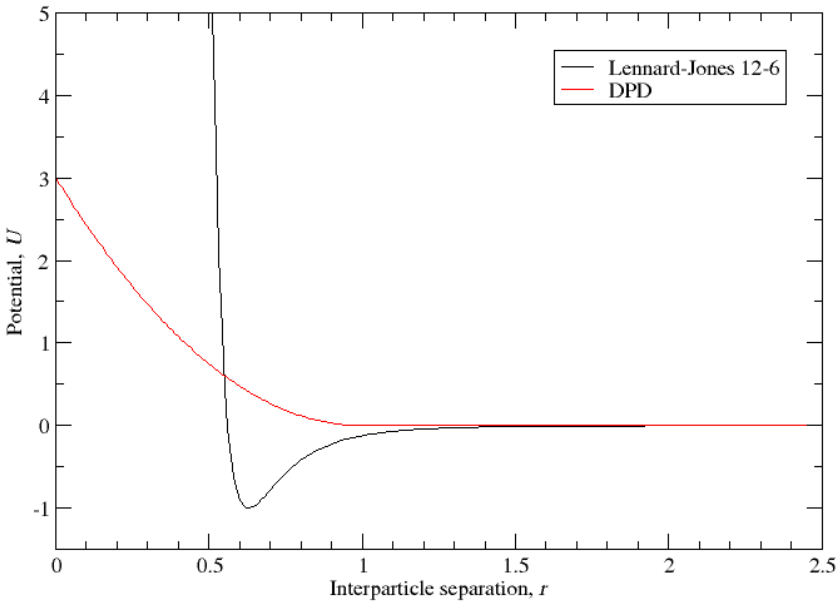
$$\phi^{GW}(r_{ij}) = \begin{cases} \frac{1}{2}A_{ij}r_c\left(1-\frac{r_{ij}}{r_c}\right)^2, & r_{ij} < r_c \\ 0, \quad . & r_{ij} \geq r_c \end{cases}$$

  - As $r_{ij} \to 0$, $\phi \to \frac{1}{2}A_{ij}r_c$
  - $\phi \geq 0$ for all values of r_{ij}

- cf. Lennard-Jones potential:

$$\phi^{LJ}(r_{ij}) = 4\epsilon_{ij}\left[\left(\frac{\sigma_{ij}}{r_{ij}}\right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}}\right)^{6}\right]$$

  - As $r_{ij} \to 0$, $\phi \to \infty$
  - $\phi_{min} = -\epsilon_{ij}$ at $r_{ij} = \sqrt[6]{2}\sigma_{ij}$
  - In practice, $r_c \geq 2.5\sigma_{ij}$



$A_{ij} = 6, r_c = 1$
$\epsilon_{ij} = 1, \sigma_{ij} = 0.55$

Note that $\mathbf{F}_{ij}^{C} = -\nabla\phi = -\frac{\partial\phi}{\partial r_{ij}}\hat{\mathbf{r}}_{ij}$

DPD algorithm

- Forces are:
  - Momentum conserving ← Theoretically most important
  - Angular momentum conserving
  - Pairwise
  - Central
- Soft potentials appealing since:
  - Interactions are local and finite-ranged
  - Longer time steps possible than for classical MD
    - Equilibria can be reached more quickly/directly

Fokker-Planck formulation

- Determines relationship between dissipative and random forces[1]
- Start with stochastic differential (Langevin) equations:

$$d\mathbf{r}_i = \mathbf{v}_i dt = \frac{\mathbf{p}_i}{m_i} dt$$

$$d\mathbf{p}_i = \left[\sum_{j \neq i} F_{ij}^C(r_{ij}) - \sum_{j \neq i} \gamma w^D(r_{ij}) (\hat{\mathbf{r}}_{ij} \cdot \mathbf{v}_{ij}) \hat{\mathbf{r}}_{ij} \right] dt + \sum_{j \neq i} \sigma w^R(r_{ij}) \hat{\mathbf{r}}_{ij} dW_{ij}$$

  - where $dW_{ij} \left(= dW_{ji}\right) \equiv \theta_{ij} dt$ is an increment of a Wiener (Brownian) process and

$$dW_{ij} dW_{i'j'} = (\delta_{ii'}\ \delta_{jj'} + \delta_{ij'} \delta_{ji'}) dt$$

[1] Español and Warren, *EPL* **30**, 191–196 (1995)

Fokker-Planck formulation

- Liouville's equation: $\frac{\partial \rho}{\partial t} = -\left(\frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} + \mathbf{F} \cdot \nabla_{\mathbf{p}}\right)\rho = -i\mathbf{L}\rho$

- Can reformulate SDEs as a two-part Liouville operator (Fokker-Planck formulation):

$$\mathbf{L} = \mathbf{L}_C + \mathbf{L}_D$$

$$\mathbf{L}_C = -\left[\sum_i \frac{\mathbf{p}_i}{m}\frac{\partial}{\partial \mathbf{r}_i} + \sum_{i,j\neq i} \mathbf{F}_{ij}^C(r_{ij})\frac{\partial}{\partial \mathbf{p}_i}\right]$$

Conservative (Hamiltonian) term

$$\mathbf{L}_D = \sum_i \frac{\partial}{\partial p_i}\left[\gamma w^D(r_{ij})(\hat{\mathbf{r}}_{ij} \cdot \mathbf{v}_{ij}) + \frac{\sigma^2}{2}\left(w^R(r_{ij})\right)^2 r_{ij}\left(\frac{\partial}{\partial \mathbf{p}_i} - \frac{\partial}{\partial \mathbf{p}_j}\right)\right]$$

Second derivative dissipative/ stochastic term

Fokker-Planck formulation

- At a steady state: $\frac{\partial \rho^{eq}}{\partial t} = \mathbf{L}\rho^{eq} = 0$
- For a conservative (Hamiltonian) system, solution is:

$$\rho^{eq} = \frac{1}{Z}\exp\left(-\frac{1}{\mathrm{k}_B T}\left[\sum_i\left(\frac{p_i^2}{2m_i} + \frac{1}{2}\sum_j \phi(r_{ij})\right)\right]\right)$$

- This solution must hold when dissipation is included!
- Therefore: $\mathbf{L}_D\rho^{eq} = 0$

$$w^D(r_{ij}) = \left(w^R(r_{ij})\right)^2$$
$$\sigma^2 = 2k_B T\gamma$$

Under these conditions, dissipative and random forces act as system thermostat and preserve hydrodynamics

Fokker-Planck formulation

- Possible to link dissipative/random forces with macroscopic Navier-Stokes fluid equations[1]
  - Ignoring conservative interactions, can derive an approximate Fokker-Planck-Boltzmann equation for evolution of particles (based on distribution functions)
  - Can then apply Chapman-Enskog analysis to (eventually) attain Navier-Stokes equations
  - Definitions of pressure tensors lead to relationships between dissipative force parameter, fluid viscosity and self-diffusivity

[1] Marsh *et al.*, *Phys Rev E* **56**, 1676–1691 (1997)

DPD fluid

- Viscosity and self-diffusivity
  - Normally assume screening function is

$$w^R(r_{ij}) = \begin{cases} 1 - r_{ij}/r_c, & r_{ij} < r_c \\ 0, & r_{ij} \geq r_c \end{cases}$$

  - Above Fokker-Planck analysis leads to following expressions based on kinetic and dissipative pressure tensors[1,2]:

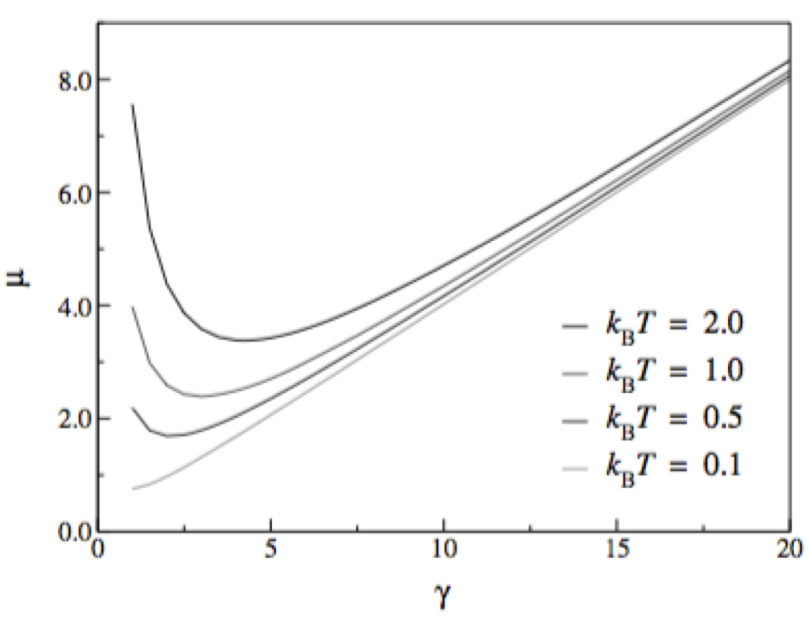
$$\nu \approx \frac{45 k_B T}{4\pi\gamma\rho r_c^3} + \frac{2\pi\gamma\rho r_c^5}{1575}$$

$$D \approx \frac{45 k_B T}{2\pi\gamma\rho r_c^3}$$

[1] Marsh *et al.*, *Phys Rev E* **56**, 1676–1691 (1997)
[2] Visser *et al.*, *J Chem Phys* **214**, 491–504 (2006)

DPD fluid

- Viscosity and self-diffusivity
  - Ratio of kinematic viscosity/diffusivity (Schmidt number) of order 1
    - OK for gases, too low for liquids?
    - Higher dissipative force parameters may make simulations unstable
  - May need to consider alternative pairwise thermostats for higher viscosities/liquids[1]



[1] Lowe, *EPL* **47**, 145–151 (1999)

DPD fluid

- Equation of state
  - Derived from virial theorem:

$$p = \rho k_B T + \frac{1}{3V}\left\langle \sum_{j \neq i} \mathbf{r}_{ij} \cdot \mathbf{F}_{ij}^{C} \right\rangle$$

$$= \rho k_B T + \frac{2\pi\rho^2}{3} \int_0^{r_c} r f(r) g(r) r^2 \, dr$$

Scalar form of conservative force

Radial distribution function

  - For ‘standard DPD’ conservative force (single component), when $\rho > 2$:

$$p = \rho k_B T + \alpha A \rho^2. \qquad (\alpha \approx 0.101 r_c^4)$$

Applying DPD for simple fluids

- Conservative force parameters
  - Values for a single component can be determined from fluid compressibility and equation of state:

$$\kappa^{-1} = \frac{V_{bead}}{k_B T \kappa_T} = \frac{1}{k_B T}\left(\frac{\partial p}{\partial \rho}\right)_T = 1 + \frac{2\alpha A \rho}{k_B T}$$

Dimensionless form of compressibility: value depends on real volume of a DPD bead

  - e.g. for water (one molecule per bead), $\kappa^{-1} \approx 16$ and:

$$A \approx \frac{75 k_B T}{\rho}$$

Applying DPD for simple fluids

- Conservative force parameters
  - Liquid-liquid interactions can be modelled by tuning A_{ij} between different species
  - Parameters can be mapped onto Flory-Huggins solution theory for polymer solutions (for $A_{ii} = A_{jj}$)[1]:

Represents excess free energy of mixing

$$\chi_{ij} = \frac{2\alpha\rho\left(A_{ij} - A_{ii}\right)}{k_B T}$$

$$\chi_{ij} \propto \Delta A_{ij}$$

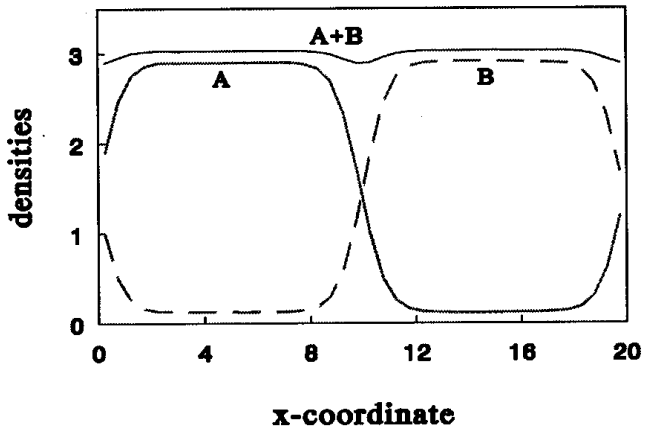
[1] Groot and Warren, *J Chem Phys* **107**, 4423–4435 (1997)

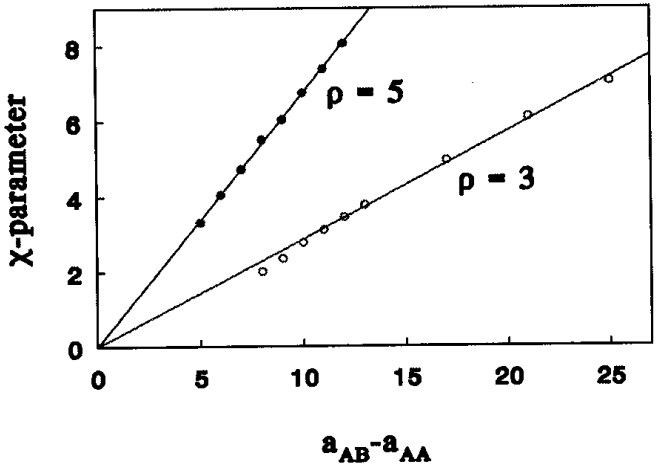
Applying DPD for simple fluids

- Conservative force parameters
  - Proportionality constant for given particle density can be determined from series of monomer simulations (finding local volume fraction ϕ_i):

$$\chi_{ij} = \frac{\ln\left[\frac{1-\phi_i}{\phi_i}\right]}{1-2\phi_i}$$

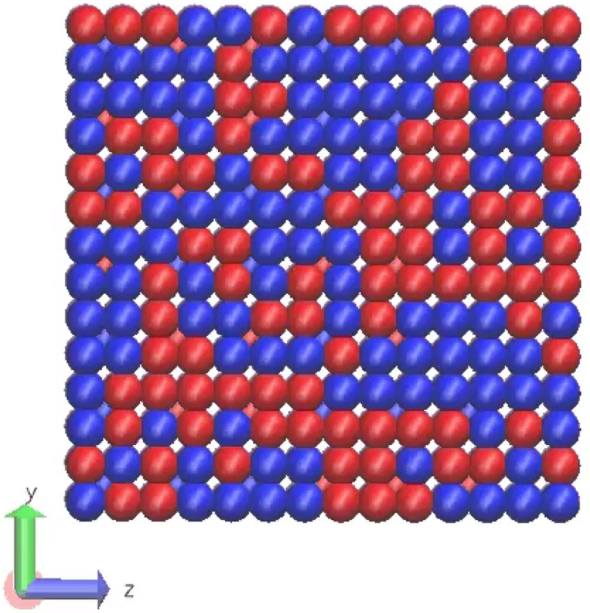
  - Proportionality constants hold for wide range of solubilities





Example: fluid/fluid separation

- Carried out using DL_MESO
  - Two species: 1500 beads each, initially randomly distributed
  - Bead masses and sizes identical
  - Interaction parameter between species: $A_{aa} = A_{bb}, A_{ab} = 4A_{aa}$
  - Separation complete after ~20 DPD time units

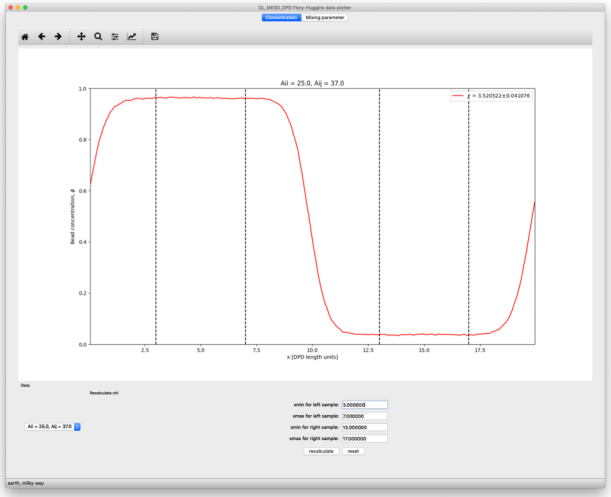


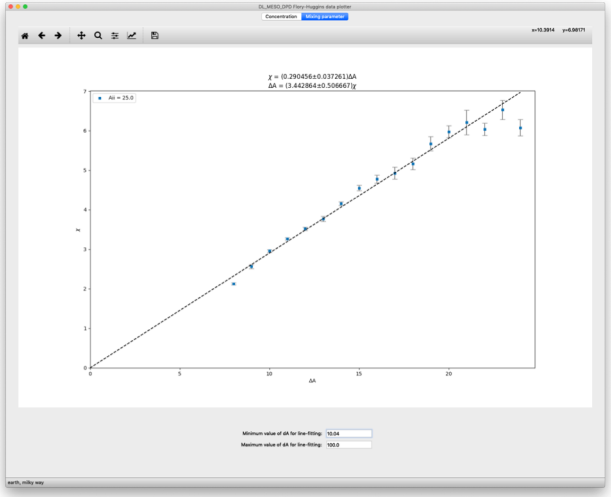
DPD Practical Exercise 1

- Demonstrate how A_{ij} relates to χ_{ij}
- Carry out simulations of separating beads with various A_{ij} values
- Find volume fractions of one component in bulk phases (ϕ_i) and work out χ_{ij}:

$$\chi_{ij} = \frac{\ln\left[\frac{1-\phi_i}{\phi_i}\right]}{1-2\phi_i}$$

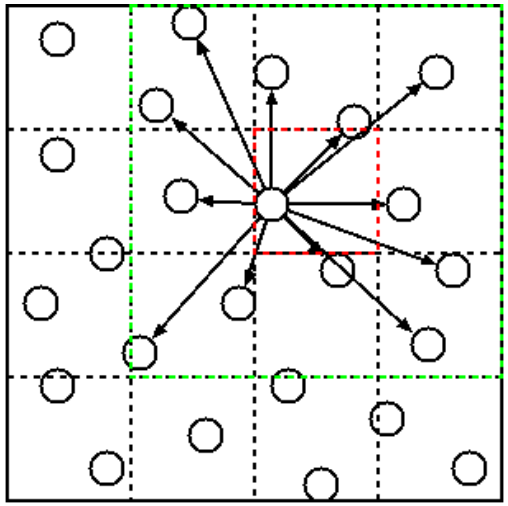
- Practical aspects:
  - Compiling DL_MESO's DPD code
  - Python scripts to automate calculations and plot χ_{ij} values
- Start from drfaustroll.gitlab.io/cs2019 (Day 4: Introduction to Dissipative Particle Dynamics)





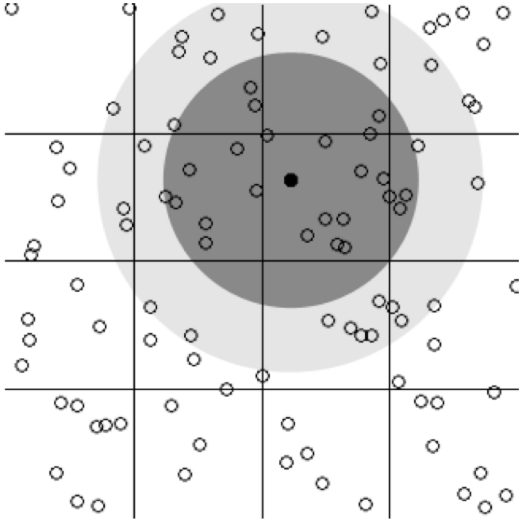
DPD calculations

- Pairwise forces
  - Only pairs within interaction cutoff need to be considered; reduces number of pairs to search
  - Possible strategies to find particle pairs:



Linked-list cells

Verlet lists



Lists might need updating frequently for systems with flow

DPD calculations

- Random forces
  - Fluctuations obtainable from Gaussian random numbers of zero mean, unity variance:

$$\theta_{ij} = \xi_{ij} \Delta t^{-\frac{1}{2}}$$

  - Most random number generators give uniformly distributed numbers ($0 \leq u < 1$), so either:
    - Convert pairs using e.g. Box-Muller transforms
    - Use central limit theorem to use generated numbers directly[1], i.e.

$$\xi_{ij} \approx \sqrt{12}\left(u_{ij} - \frac{1}{2}\right)$$

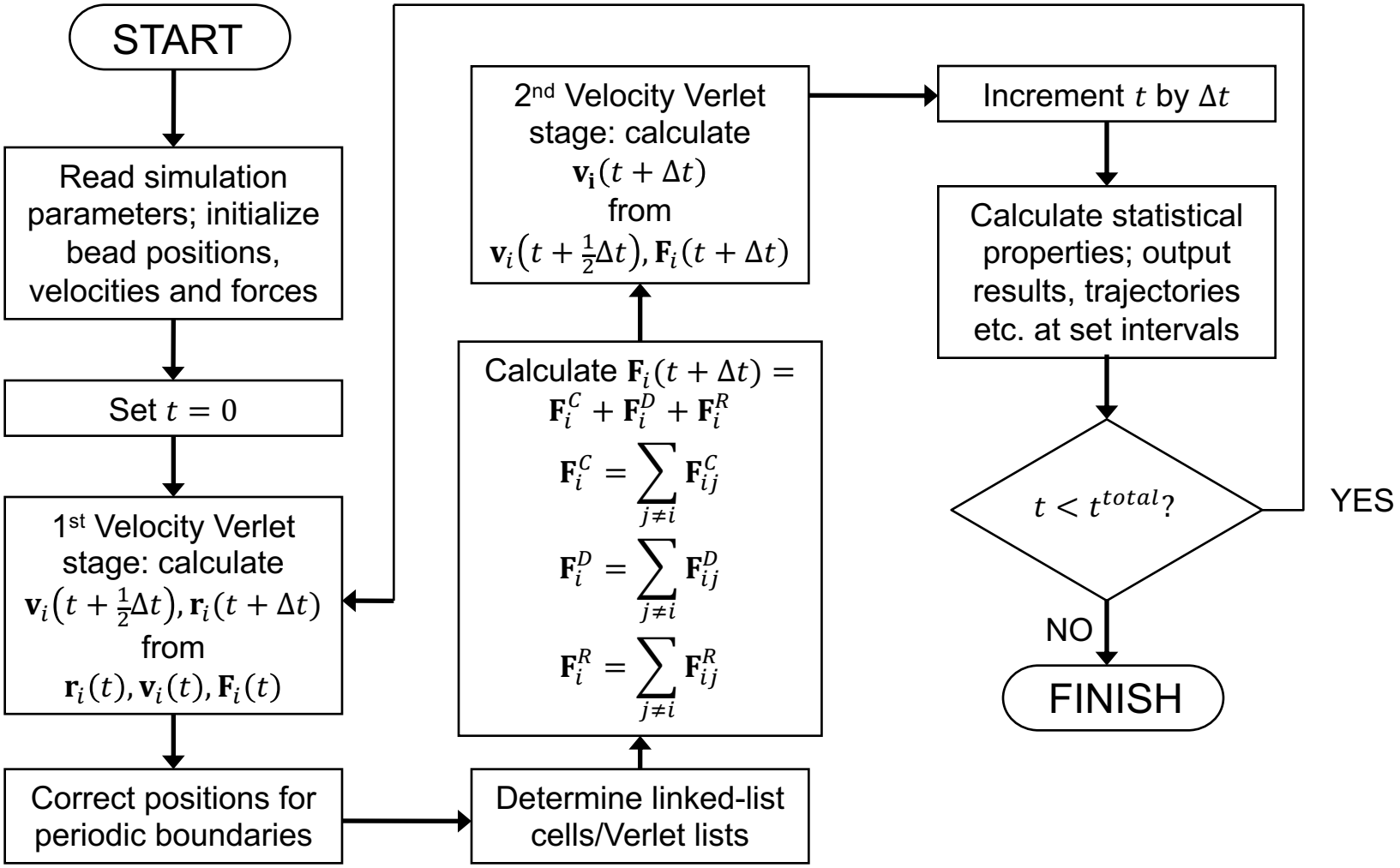
Gives 'statistically similar' results to true Gaussian random numbers[2]

[1] Dünweg and Paul, *Int J Mod Phys C* **2**, 817–827 (1991)
[2] Groot and Warren, *J Chem Phys* **107**, 4423–4435 (1997)

DPD calculations

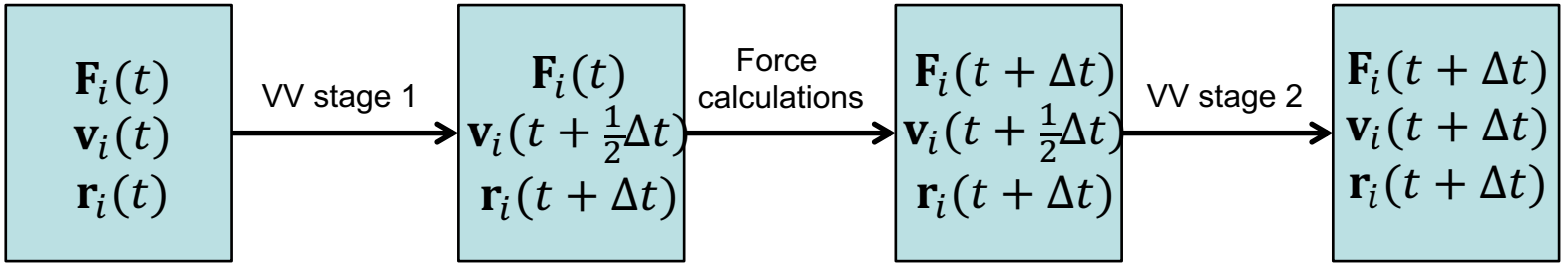
- Force integration (Velocity Verlet)
  - Mid-step velocity: $\mathbf{v}_i\left(t+\frac{1}{2}\Delta t\right)=\mathbf{v}_i(t)+\frac{\Delta t}{2}\frac{\mathbf{F}_i(t)}{m_i}$
  - Positions: $\mathbf{r}_i(t+\Delta t)=\mathbf{r}_i(t)+\mathbf{v}_i\left(t+\frac{1}{2}\Delta t\right)\Delta t$
    - Move beads
  - Calculate forces $\mathbf{F}_i(t)\rightarrow\mathbf{F}_i(t+\Delta t)$
  - Final velocity: $\mathbf{v}_i(t+\Delta t)=\mathbf{v}_i\left(t+\frac{1}{2}\Delta t\right)+\frac{\Delta t}{2}\frac{\mathbf{F}_i(t+\Delta t)}{m_i}$

DPD calculations



DPD calculations

- Force integration problem



  - Dissipative force needed at end of time step, but calculated using mid-step velocities:

$$\mathbf{F}_{ij}^{D}(t+\Delta t) = -\gamma w^{D}\left(r_{ij}(t+\Delta t)\right)\left(\hat{\mathbf{r}}_{ij}(t+\Delta t)\cdot\left(\mathbf{v}_j\left(t+\tfrac{1}{2}\Delta t\right)-\mathbf{v}_i\left(t+\tfrac{1}{2}\Delta t\right)\right)\right)\hat{\mathbf{r}}_{ij}(t+\Delta t)$$

    - Error carried forward to next time step
    - Results in temperature offset that increases with time step size

DPD calculations

- Force integration problem
  - Solution 1: Estimate end-step velocity for dissipative force calculations[1]

$$\tilde{\mathbf{v}}_i(t+\Delta t) = \mathbf{v}_i(t) + \lambda \Delta t \frac{\mathbf{F}_i(t)}{m_i}$$

    - Requires empirical factor λ; optimal value depends on system being modelled
  - Solution 2: Recalculate dissipative forces at end of time step (DPD Velocity Verlet)[2]
    - Can be made self-consistent by iterating dissipative forces/velocities

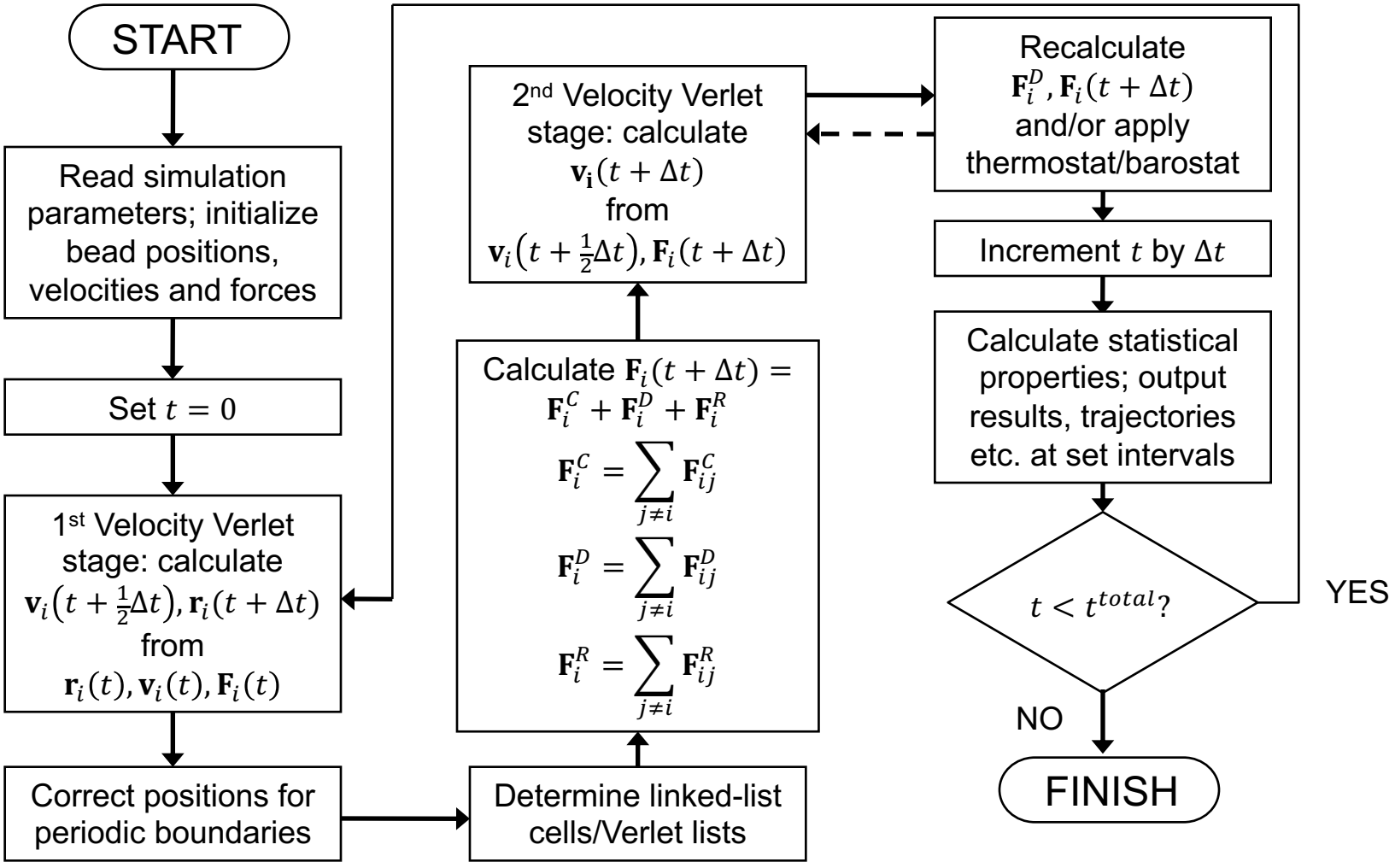
[1] Groot and Warren, *J Chem Phys* **107**, 4423–4435 (1997)
[2] Nikunen *et al.*, *Comp Phys Comms* **153**, 407–423 (2003)

DPD calculations

- Force integration problem
  - Solution 3: Apply exponential propagators for dissipative/random forces (e.g. Shardlow splitting[1])
    - Uses velocities at start of time step
    - Conservative forces still integrated with Velocity Verlet
  - Solution 4: Use an alternative pairwise thermostat (e.g. Lowe-Andersen[2], Peters[3])
    - Applies pairwise velocity corrections at end of time step after integration of conservative forces
    - Lowe-Andersen thermostat (and its variants) can increase Schmidt number

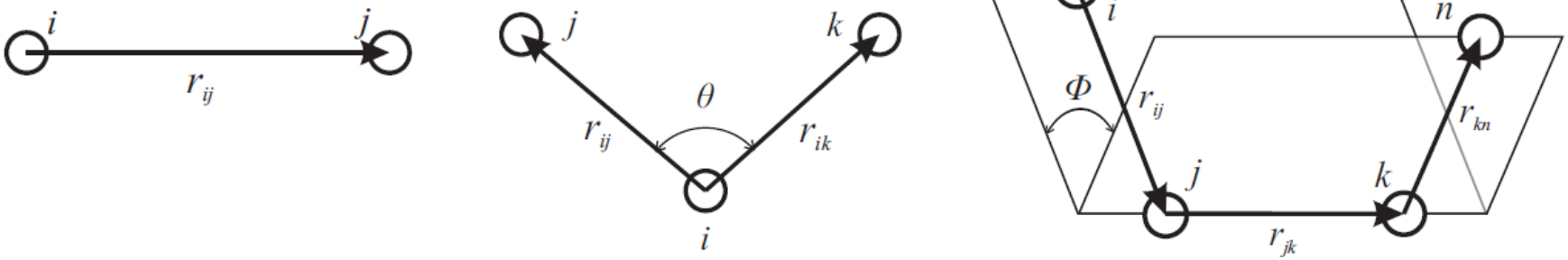
[1] Shardlow, *SIAM J Sci Comput* **24**, 1267–1282 (2003)
[2] Lowe, *EPL* **47**, 145–151 (1999)
[3] Peters, *EPL* **66**, 311–317 (2004)

DPD calculations



Applying DPD for complex fluids

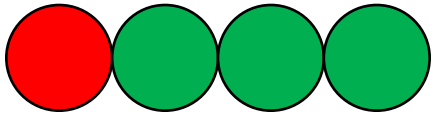
- DPD beads can be bonded together to form molecular chains
  - Stretching, angles, dihedrals



  - Used to give vestige (impression) of molecular structures
  - Vastly extends range of DPD simulations (e.g. polymers, colloids, biological)

Example: vesicle formation[1]

- Four-bead amphiphilic molecule:

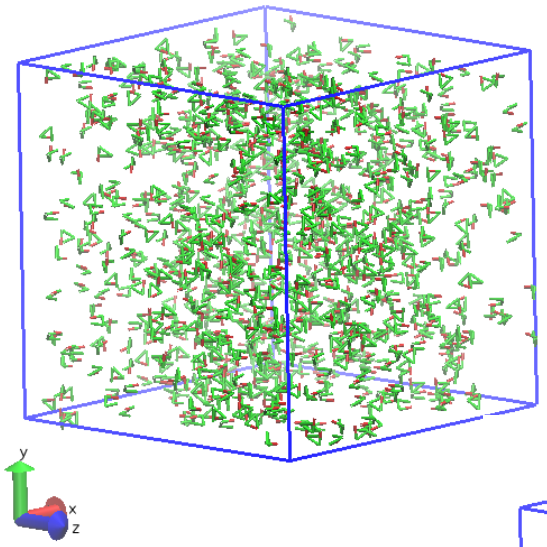


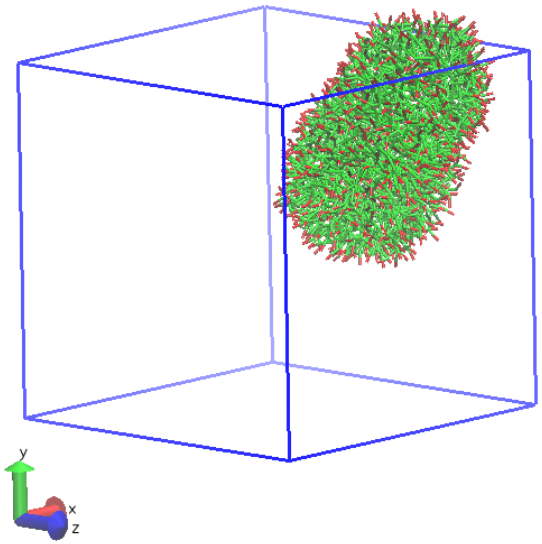
A = hydrophilic, B = hydrophobic

  - Bonded by stiff harmonic springs with equilibrium length = cut-off radius
- Single equally-sized beads for water
- Interactions: $A_{AA} = A_{BB} = A_{WW} = A_{AW} = 25, A_{AB} = A_{BW} = 40$
- Simulation properties: $k_B T = 0.25, \gamma = 6.7, \Delta t = 0.05$
  - Simulation carried out using DL_MESO in parallel on 96 cores of HECToR (former UK national supercomputer)

[1] Yamamoto *et al.*, *J Chem Phys* **116**, 5842–5849 (2002)

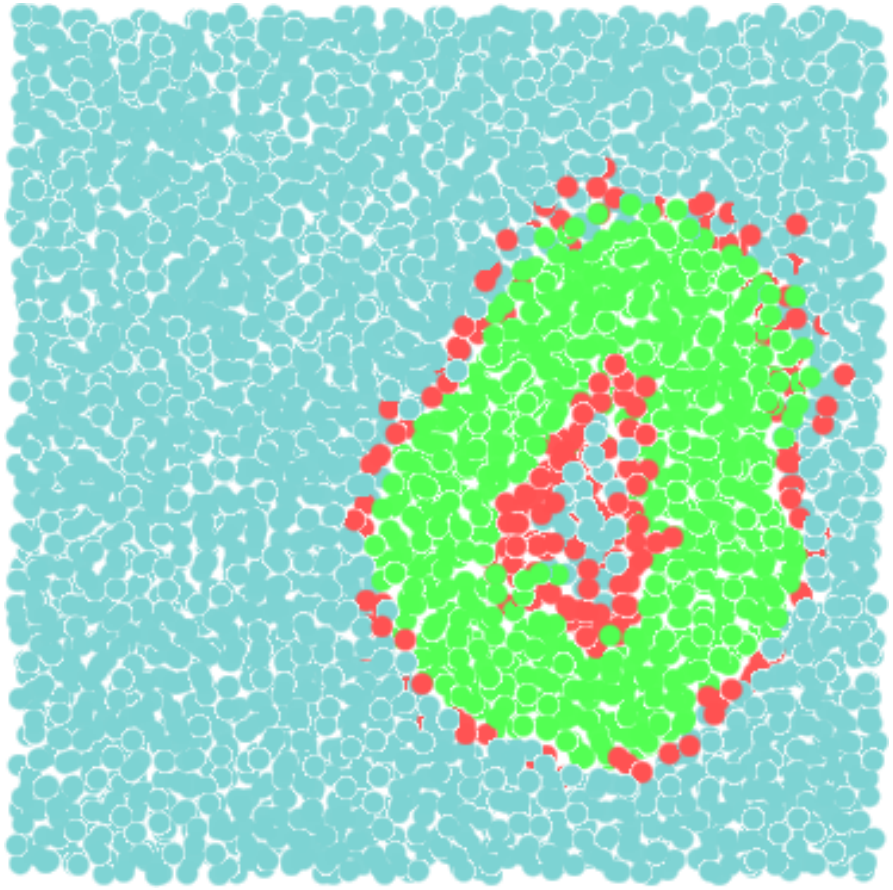
Example: vesicle formation





- Spontaneous vesicle formation
  - 41,472 beads in total (9.7 vol% amphiphilic molecules)
    - Molecules initially placed randomly in box
  - 1,000,000 time steps (50,000 time units)
  - Water omitted to show amphiphiles more clearly

Example: vesicle formation



- Spontaneous vesicle formation
  - Cross-section at last time step
  - Vesicle formed from large oblate micelle
  - Water incorporated into vesicle
  - Applications include e.g. drug delivery

Applying DPD for complex fluids

- Charged particles
  - Like classical MD, possible to apply electrostatic interactions between DPD beads
  - Charged interactions act over large length scales
    - Need some method to deal with long-range interactions, e.g. Ewald sum, Particle-Particle Particle-Mesh (P^3M)
  - Cannot always use point charges with DPD beads!
    - 'Standard' conservative interactions (soft, repulsive) are often insufficiently strong to prevent ion collapse[1]
    - May need to smear out charges over finite volumes

[1] Terrón-Mejía *et al.*, *J Phys: Cond Mat* **28**, 425101 (2016)

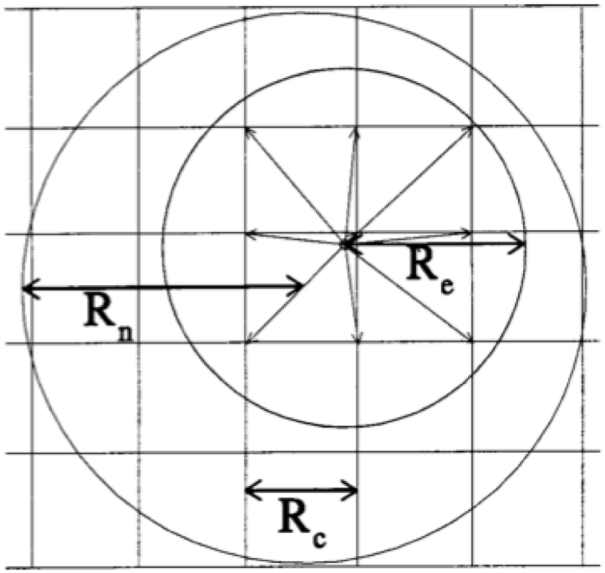
Applying DPD for complex fluids

- Charged particles: P^{3}M approach[1]
  - Construct grid for charge concentrations ρ^*, smearing out charges with function $f(\mathbf{r}_i)$
  - Solve Poisson equation (iteratively) to give electrostatic field:

$$\nabla \cdot (p(r)\nabla\psi) = -\left(\frac{e^2\rho r_c^2}{k_B T \epsilon_0 \epsilon_r}\right)\rho^* = -\Gamma\rho^*$$

  - Spatial differentiation of electrostatic field (summed over grid) gives force on ion:

$$\mathbf{F}_i^E(r_i) = -q_i \sum_j f_j(\mathbf{r}_i)\nabla\psi(\mathbf{r}_j)$$



[1] Groot, *J Chem Phys* **118**, 11265–11277 (2003)

Applying DPD for complex fluids

- Charged particles: Ewald sum approaches
  - Modify Ewald sums to include charge smearing function in pairwise electrostatic potential, i.e.

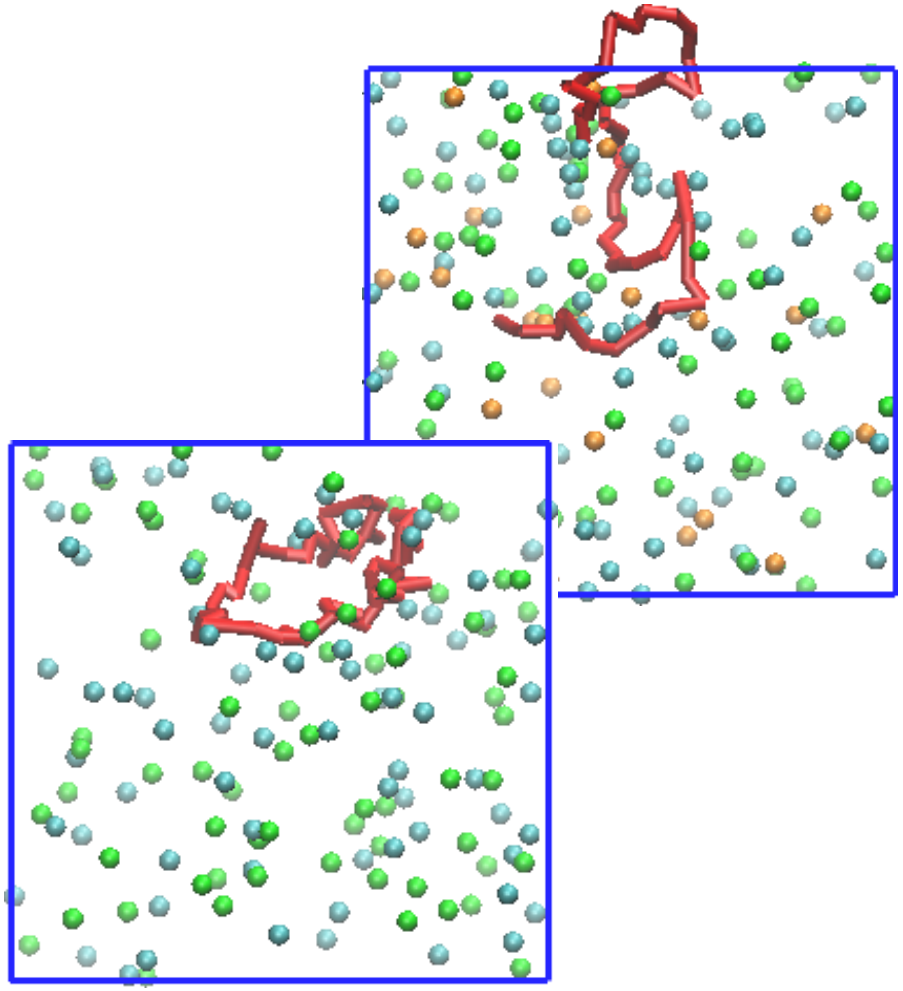
$$U_{ij}^{E} = \frac{\Gamma q_i q_j}{4\pi r_{ij}}\left[1 - f(r_{ij})\right]$$

  - Smearing assumed to only affect real-space part:
    - Often needs sufficiently large real-space cutoff to converge with Coulombic (point charge) distribution in reciprocal space
    - Reciprocal space parts carried out as usual
  - Commonly used smearing functions:
    - Exponential (Slater-type) smearing[1]: $f(r) = (1 + \beta r)e^{-2\beta r}$
    - Gaussian charge smearing[2]: $f(r) = \mathrm{erfc}\left(\frac{r}{2\sigma_G}\right)$

Can tune Ewald sum parameter α to coincide with σ_G: real-space terms then reduce to zero

[1] González-Melchor *et al.*, *J Chem Phys* **125**, 224107 (2006)
[2] Warren *et al.*, *J Chem Phys* **138**, 204907 (2013)

Example: polyelectrolyte[1]

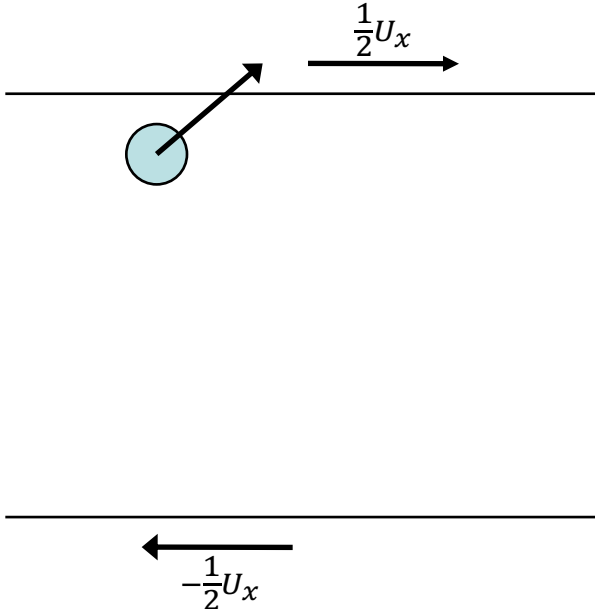


- Polyelectrolyte in salt solution
  - Modelled with Ewald sum and Slater-type smearing
  - Hydrophobic polymer of 50 charged beads
  - Additional charged beads for salt, anions and counterions
  - Can measure how far molecule stretches out using radius of gyration

[1] González-Melchor *et al.*, *J Chem Phys* **125**, 224107 (2006)

Boundary conditions

- Most DPD simulations involve bulk systems at rest (periodic BCs)
- Can also subject them to flow fields, e.g. uniform shear (Lees-Edwards BCs)[1]



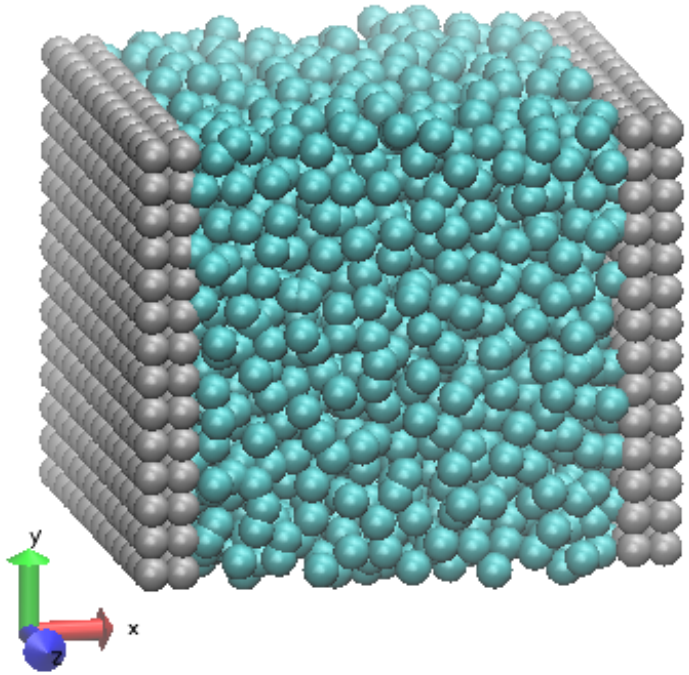
If bead crosses upper boundary at time t, re-introduced at lower boundary with x-coordinate shifted by $-U_x t$ and x-component of velocity decreased by U_x. (Vice versa if bead crosses lower boundary.)

Relative velocity between particle pairs across boundary adjusted to remove wall speed difference[2]

[1] Lees and Edwards, *J Phys C* **5**, 1921–1928 (1972)
[2] Leimkuhler and Shang, *J Comput Phys* **324**, 174–193 (2016)

Boundary conditions

- Possible to represent solid walls by planar reflections[1] or frozen DPD beads
  - Can produce artificial density/temperature fluctuations at and near walls
  - Often reduced by combination of both methods
- Used to model solid/liquid interactions
  - e.g. wetting, polymer brushes



[1] Warren *et al.*, *Phil Trans Roy Soc Lond A* **361**, 665–676 (2003)

Thermodynamics and DPD

- Standard form of conservative interaction gives quadratic equation of state:

$$p = \rho k_B T + \alpha A \rho^2$$

  - Possible to map fluid compressibility or pressure onto EOS, but not both!
  - Not flexible enough to model thermodynamics of real systems in great detail
  - Note that conservative force currently only depends on bead pair separation r_{ij}

Thermodynamics and DPD

- Many-body (density dependent) DPD[1,2]
  - Suppose each DPD particle has a localised density ρ_i and associated free energy (sum of bulk, mixing and excess components):

$$\psi(\rho_i) = \psi^{bulk}(\rho_i) + \psi^{mixing}(\rho_i) + \psi^{excess}(\rho_i)$$

Associated with ideal gas of force-free particles

  - Free energy density of system can be written:

$$F = \int \rho(r)\psi\big(\rho(r)\big)dr \xrightarrow{\Delta} \sum_i \rho_i \psi(\rho_i)$$

[1] Pagonabarraga and Frenkel, *J Chem Phys* **115**, 5015–5026 (2001)
[2] Trofimov *et al.*, *J Chem Phys* **117**, 9383–9394 (2002)

Thermodynamics and DPD

- Many-body (density dependent) DPD
  - Ideal gases have dissipation and mixing, thus excess free energy must be related to conservative interactions; tentatively given as

$$\mathbf{F}_i^C = -\nabla\left(\sum_{j\neq i}\rho_j\psi^{excess}(\rho_j)\right)$$

  - Local density approximated as sum of weight functions between bead pairs:

$$\rho_i = \sum_{j\neq i} w(r_{ij}) = \sum_{j\neq i}\frac{15}{2\pi}\left(1-\frac{r_{ij}}{r_d}\right)^2$$

This form of weight function can recover 'standard' DPD

Thermodynamics and DPD

- Many-body (density dependent) DPD
  - Many-body conservative interaction forces can be rewritten in pairwise additive form:

$$\mathbf{F}_i^C = -\sum_{j \neq i} \left(\psi'^{excess}(\rho_i) + \psi'^{excess}(\rho_j) \right) w^C(r_{ij}) \hat{\mathbf{r}}_{ij}$$

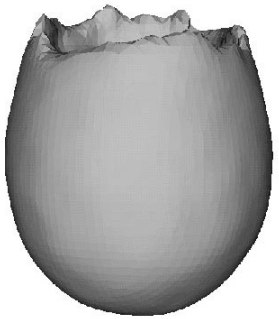
  - Excess free energy form can be chosen to give required equation of state:

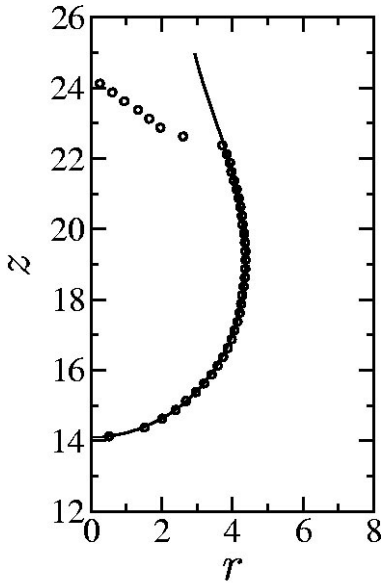
$$p \approx \rho k_B T + \rho^2 \left\langle \frac{\partial \psi^{excess}}{\partial \rho} \right\rangle \left(-\frac{1}{3} \iiint_D r_{ij} w^C(r_{ij}) g(r_{ij}) dx\, dy\, dz \right)$$

Spatial integral over system volume

Thermodynamics and DPD

- Many-body (density dependent) DPD
  - Attractive interactions can be included, e.g. vapour-liquid systems[1]





Standard DPD force augmented by many-body (density-dependent) term:

$$\mathbf{F}_i^C = \sum_{j \neq i} \left[A_{ij} w^C(r_{ij}, r_c) + B(\rho_i + \rho_j) w^C(r_{ij}, r_d) \right] \hat{\mathbf{r}}_{ij}$$

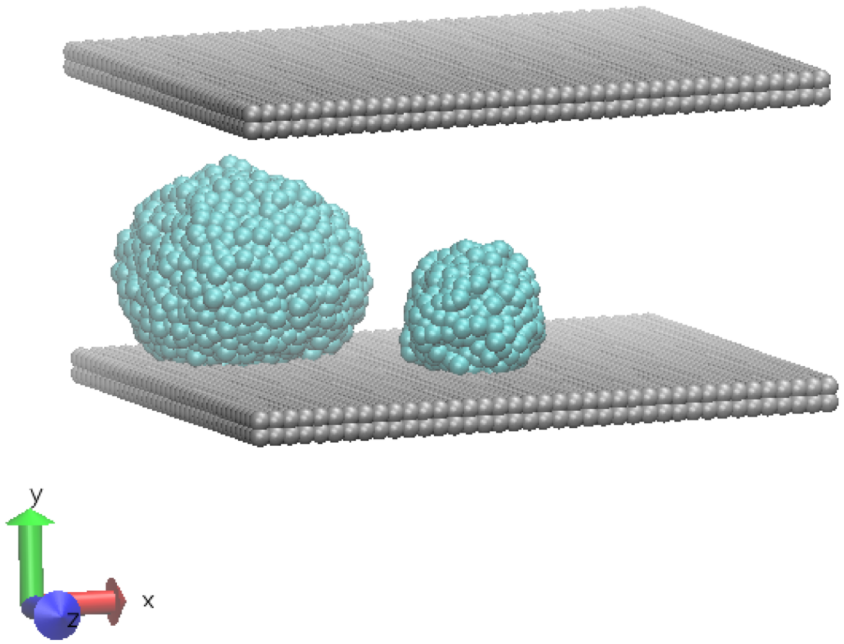
which gives a van der Waals-type (cubic) equation of state when $A_{ij} < 0$ and $B > 0$:

$$p = \rho k_B T + \alpha A_{ij} \rho^2 + 2\alpha B r_d^4 (\rho^3 - c\rho^2 + d)$$

[1] Warren, *Phys Rev E* **68**, 066702 (2003)

Example: drop on surface

- Formation of water drops on hydrophobic surface
  - Two-parameter many-body DPD: $A_{SW} > A_{WW}$
  - Frozen bead walls with bounce-back reflections
  - Interaction between water and wall more repulsive
  - Constant gravitational force on system



Molecular dynamics and DPD

- DPD parameters can be fitted on bottom-up basis, e.g. from MD simulations[1]
  - Conservative parameters: Inverse Monte Carlo method using radial distribution function

$$g(r) = \exp\left(-\frac{\phi(r)}{k_B T}\right) + f(\rho, r)$$

Second term negligible for low density gases

  - Dissipative/random parameters: Best fit to velocity autocorrelation function (VAF)

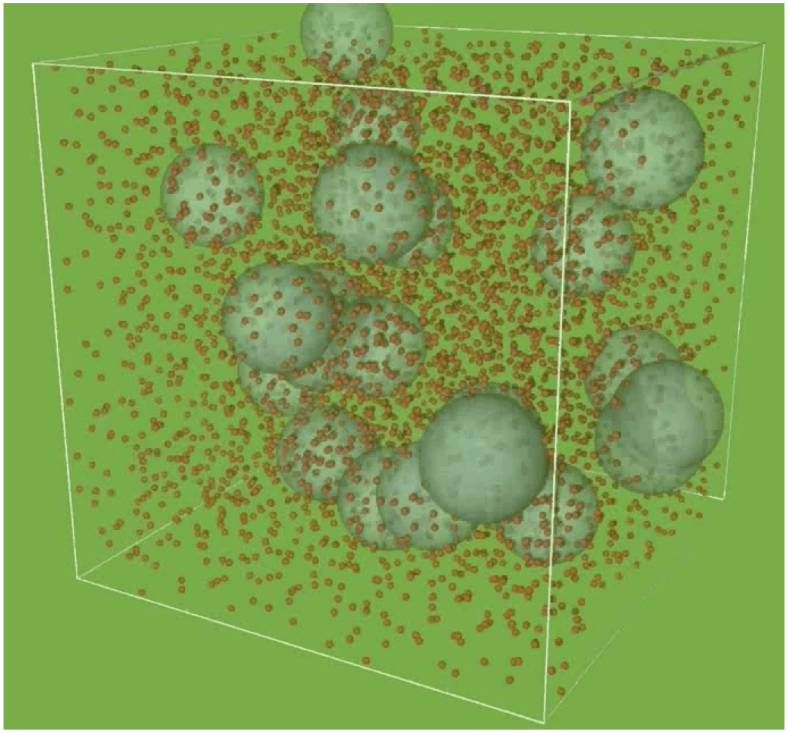
$$C(t) = \langle \mathbf{v}_i(0) \cdot \mathbf{v}_i(t) \rangle$$

Related to diffusion: $D = \frac{1}{3}\int_0^\infty C(t)\, dt$

[1] Lyubartsev *et al.*, *Soft Mat* **1**, 121–137 (2002)

Example: fitting DPD to MD

- Obtaining dissipative parameters
  - Initial MD simulation of Lennard-Jones fluid (red particles)
  - DPD beads (transparent spheres) applied afterwards: follow average dynamics of LJ fluid
  - DPD random force parameter can be obtained by comparing VAFs of LJ particles and DPD beads



Video courtesy: Vlad Sokhan

Molecular dynamics and DPD

- MD simulations can also be used to carry out top-down fitting approach for DPD parameters
  - e.g. calculate mixing excess free energy between two components from constant-pressure atomistic MD simulations of mixture and pure components [1]

$$\Delta G^{mix} = U_{ij} - \left(U_i + U_j\right)$$

  - Mixing free energy related to Flory-Huggins χ-parameter:

$$\chi_{ij} = \frac{\Delta G^{mix} V_{ref}}{RT\phi_i\phi_j V}$$

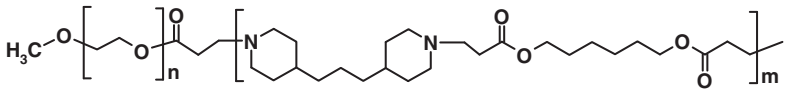
  - Use determined relationship between χ_{ij} and A_{ij}, e.g.

$$A_{ij} \approx A_{ii} + 3.50\chi_{ij}$$

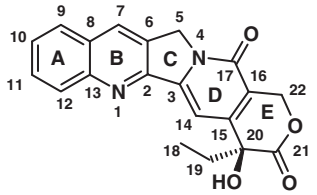
[1] Akkermans, *J Chem Phys* **128**, 244904 (2008)

Example: drug loading/release[1]

- Copolymer PAE-PEG with water and camptothecin (CPT)



PEG Poly(β-amino ester)(PAE)



Camptothecin (CPT)

- Conservative force parameterisation
  - Determined χ-parameters for PAE, PEG and CPT from MD simulations (see previous slide) with COMPASS force field
  - Modelling at two pH levels (7.4 and 6.4): using hydrated form of PAE with chloride ions in MD simulations for more acidic system

[1] Luo and Jiang, *J Control Release* **162**, 185–193 (2012)

Example: drug loading/release

- Two sets of conservative force parameters A_{ij}

	H_2O	CPT	PEG	PAE
H_2O	25.00	48.22	26.05	125.23
CPT		25.00	22.28	22.49
PEG			25.00	46.81
PAE				25.00

Drug loading: pH ≈ 7.4

Can bond PAE and PEG beads together to form copolymers (e.g. 28 PAE beads and 10 PEG beads represent PAE_{12580}-PEG_{4850})

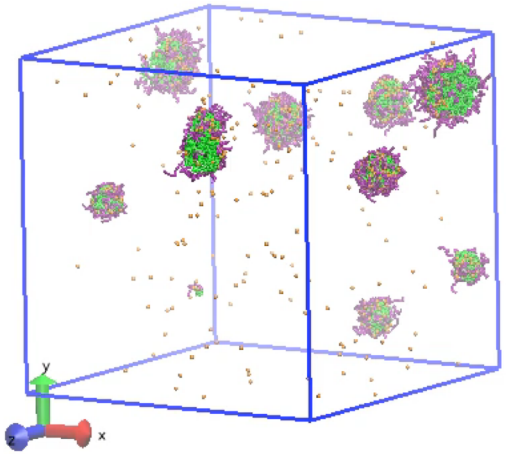
	H_2O	CPT	PEG	PAE
H_2O	25.00	48.22	26.05	19.10
CPT		25.00	22.28	82.04
PEG			25.00	23.63
PAE				25.00

Drug release: pH ≈ 6.4

PAE replaced with hydrated form: note changes to hydrophobicity between PAE and water/CPT/PEG

Example: drug loading/release

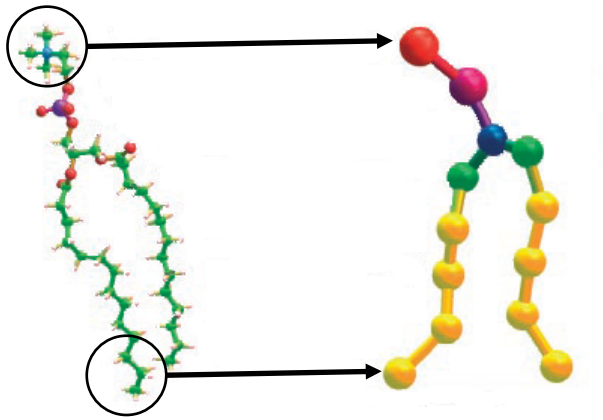
- DPD simulations with DL_MESO
  - Ran drug loading simulation from initial randomly distributed state over 900ns (1 million timesteps)
    - PAE-PEG form vesicles that encapsulate CPT
  - Last timeframe of drug loading simulation used as initial state for drug release
    - Vesicles swell, break open and push out CPT due to drop in pH
  - Drug release simulation ran for 90ns (100,000 timesteps)
  - Loading efficiency closely matches experimental observation



Simulation courtesy: Nidhi Raj

Molecular dynamics and DPD

- Coarse-graining in DPD
  - Each DPD bead can represent:
    - An individual atom?
    - A functional group[1]
    - An entire molecule
    - A group of molecules



  - Level of coarse-graining influences interactions between DPD beads
  - No single ‘best method’ to coarse-grain or determine interactions

[1] Lopez *et al.*, *Proc Nat Acad Sci USA* **101**, 4431–4434 (2004)

Molecular dynamics and DPD

- Coarse-graining in DPD
  - Coarse-graining limit for fluids: artificial crystal formation occurs for large values of $A_{ij} > 250\ k_B T$
  - Conservative interaction strength scales with coarse-graining level:
    - Linear relationship[1,2]:

      $$A_{ij} \propto \phi$$

    - Less linear (considers that $r_c \propto \phi^{\frac{1}{3}}$)[3]:

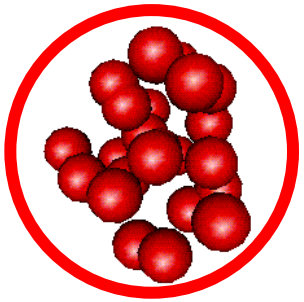
      $$A_{ij} \propto \phi^{\frac{2}{3}}$$

  - Many-body DPD can offer greater coarse-graining, but greater risk of numerical instability

[1] Groot and Rabone, *Biophys J* **81**, 725–736 (2001)
[2] Trofimov, PhD Thesis, Technical University of Eindhoven (2003)
[3] Füchslin *et al.*, *J Chem Phys* **130**, 214102 (2009)

Molecular dynamics and DPD

- Relationship between MD and DPD[1-3]
  - Can help prove DPD's hydrodynamics
  - Assume DPD bead is collection of MD particles (e.g. atoms): cell on Voronoi lattice
  - Define sampling function which can express macroscopic observables, e.g. mass, momentum



$$f_k(\mathbf{r}) = \frac{s(\mathbf{r} - \mathbf{r}_k)}{\sum_l s(\mathbf{r} - \mathbf{r}_l)}$$

$\mathbf{r}_k$ = centre of DPD bead

$s(\mathbf{r})$ = localized function, e. g. $\exp\left(-\frac{r^2}{a^2}\right)$

[1] Flekkøy and Coveney, *Phys Rev Lett* **83**, 1775–1778 (1999)
[2] Flekkøy *et al.*, *Phys Rev E* **62**, 2140–2157 (2000)
[3] De Fabritiis *et al.*, *Phil Trans Roy Soc Lond A* **360**, 317–331 (2002)

Molecular dynamics and DPD

- Relationship between MD and DPD
  - Assuming MD particles are of equal mass:
    - Mass of DPD bead: $M_k = \sum_i f_k(\mathbf{r}_i) m$
    - Momentum: $\mathbf{P}_k = \sum_i f_k(\mathbf{r}_i) m\mathbf{v}_i \equiv M_k \mathbf{V}_k$
    - Energy: $E_k^T = \sum_i f_k(\mathbf{r}_i) \left[\frac{1}{2} m v_i^2 + \frac{1}{2} \sum_{j \neq i} \phi(r_{ij}) \right]$
      $\equiv \frac{1}{2} M_k V_k^2 + E_k^P$
  - If $\sum_k f_k(r) = 1$:

$$\sum_k M_k = \sum_i m, \sum_k M_k \mathbf{V}_k = \sum_i m\mathbf{v}_i, \sum_k E_k^T = \sum_i e_i^T$$

Molecular dynamics and DPD

- Relationship between MD and DPD
  - DPD equations of motion obtained from material derivatives of macroscopic quantities

$$\dot{f}_k(\mathbf{r}_i) = \frac{Df_k}{Dt} = \frac{d}{dt}\left[\frac{s(\mathbf{r}-\mathbf{r}_k)}{\sum_l s(\mathbf{r}-\mathbf{r}_l)}\right]$$
$$= \sum_i f_{kl}(\mathbf{r}_i)[\mathbf{v}_i' \cdot \mathbf{r}_{kl} + \mathbf{r}_i' \cdot \mathbf{V}_{kl}]$$

Using quotient rule and substitutions below

$$f_{kl}(\mathbf{r}_i) = \frac{2}{a^2} f_k(\mathbf{r}_i) f_l(\mathbf{r}_i)$$

Overlap function

$$\mathbf{r}_{kl} = \mathbf{r}_k - \mathbf{r}_l$$
$$\mathbf{V}_{kl} = \mathbf{V}_k - \mathbf{V}_l$$
$$\mathbf{r}_i' = \mathbf{r}_i - \tfrac{1}{2}(\mathbf{r}_k + \mathbf{r}_l)$$
$$\mathbf{v}_i' = \mathbf{v}_i - \tfrac{1}{2}(\mathbf{V}_k + \mathbf{V}_l)$$

DPD centred parameters

Molecular dynamics and DPD

- Relationship between MD and DPD
  - DPD equations of motion obtained from material derivatives of macroscopic quantities
    - Mass:

$$\dot{M}_k = \sum_i \dot{f}_k(\mathbf{r}_i)m = \sum_i f_{kl}(\mathbf{r}_i)m[\mathbf{v}_i' \cdot \mathbf{r}_{kl} + \mathbf{r}_i' \cdot \mathbf{V}_{kl}] = \sum_i \dot{M}_{kl}$$

    - Momentum:

$$\dot{\mathbf{P}}_k = \sum_i \tfrac{1}{2}\dot{M}_{kl}\,(\mathbf{V}_k + \mathbf{V}_l) + \sum_{l,i} f_{kl}(\mathbf{r}_i)\boldsymbol{\Pi}_{\mathrm{i}}' \cdot \mathbf{r}_{kl} + \sum_{l,i} f_{kl}(\mathbf{r}_i)m\mathbf{v}_i'\mathbf{r}_i \cdot \mathbf{V}_{kl}$$

Momentum flux tensors:

$$\boldsymbol{\Pi}_i = m\mathbf{v}_i\mathbf{v}_i + \tfrac{1}{2}\sum_j \mathbf{F}_{ij}(\mathbf{r}_i - \mathbf{r}_j)$$

$$\boldsymbol{\Pi}_i' = m\mathbf{v}_i'\mathbf{v}_i' + \tfrac{1}{2}\sum_j \mathbf{F}_{ij}(\mathbf{r}_i - \mathbf{r}_j)$$

Molecular dynamics and DPD

- Relationship between MD and DPD
  - DPD equations of motion obtained from material derivatives of macroscopic quantities
    - Energy:

$$\dot{E}_k^T = \frac{d}{dt}\left(\tfrac{1}{2}M_k V_k^2\right) + \sum_l \tfrac{1}{2}\dot{M}_{kl}\left(\tfrac{1}{2}\mathbf{V}_{kl}\right)^2 + \sum_{l,i} f_{kl}(\mathbf{r}_i)\left(\mathbf{J}'_{e,i} - \tfrac{1}{2}\boldsymbol{\Pi}'_i \cdot \mathbf{V}_{kl}\right)\cdot \mathbf{r}_{kl}$$

$$+ \sum_{l,i} f_{kl}(\mathbf{r}_i)\left(e'_i - \tfrac{1}{2}m\mathbf{v}'_i \cdot \mathbf{V}_{kl}\right)\mathbf{r}'_i \cdot \mathbf{V}_{kl}$$

Energy flux tensors:

$$\mathbf{J}_{e,i} = e\mathbf{v}_i + \tfrac{1}{4}\sum_j \mathbf{F}_{ij}(\mathbf{v}_i + \mathbf{v}_j)\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

$$\mathbf{J}'_{e,i} = e\mathbf{v}'_i + \tfrac{1}{4}\sum_j \mathbf{F}_{ij}(\mathbf{v}'_i + \mathbf{v}'_j)\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

Molecular dynamics and DPD

- Relationship between MD and DPD
  - Ensemble averaging of material derivatives
    - Need to average over states of MD particles to give same properties (mass, velocity, energy) as DPD beads
    - Evolution can be represented (in terms of statistical mechanics) by average and fluctuating parts
    - Assume that average velocity of MD particles between DPD beads can be approximated by nearest neighbour information and determined from linear interpolation:

$$\langle \mathbf{v}' \rangle \approx \frac{\mathbf{r}' \cdot \mathbf{r}_{kl}}{r_{kl}^2} \mathbf{v}_{kl}$$

Molecular dynamics and DPD

- Relationship between MD and DPD
  - Ensemble averaging of material derivatives
    - Mass: $$\dot{M}_k = \sum_l \left(\langle \dot{M}_{kl} \rangle + \tilde{\dot{M}}_{kl} \right)$$
      $$\langle \dot{M}_{kl} \rangle = \sum_i m f_{kl}(\mathbf{r}_i) \langle \mathbf{r}_i' \rangle \cdot \mathbf{v}_{kl}$$
      $$\tilde{\dot{M}}_{kl} \equiv \dot{M}_{kl} - \langle \dot{M}_{kl} \rangle$$
    - Mass fluctuations visible in momentum and energy fluxes
    - Mass conservation otherwise observed

Molecular dynamics and DPD

- Relationship between MD and DPD
  - Ensemble averaging of material derivatives
    - Momentum:

$$\dot{\mathbf{P}}_k = \sum_i \tfrac{1}{2}\langle \dot{M}_{kl} \rangle (\mathbf{V}_k + \mathbf{V}_l) + \sum_l f_{kl}(\mathbf{r}_i)\langle \mathbf{\Pi}_i \rangle \cdot \mathbf{r}_{kl}$$
$$+ \sum_i f_{kl}(\mathbf{r}_i) m \langle \mathbf{v}_i' \mathbf{r}_i' \rangle \cdot \mathbf{V}_{kl} + \sum_l \tilde{\mathbf{F}}_{kl}$$

    - Fluctuating force (similar to DPD random force):

$$\tilde{F}_{kl} = \sum_i f_{kl}(\mathbf{r_i})[(\mathbf{\Pi}_i - \langle \mathbf{\Pi}_i \rangle) \cdot \mathbf{r}_{kl} + m(\mathbf{v}_i' \mathbf{r}_i' - \langle \mathbf{v}_i' \mathbf{r}_i' \rangle) \cdot \mathbf{V}_{kl}]$$
$$+ \tfrac{1}{2}\dot{\tilde{M}}_{kl}(\mathbf{V}_k + \mathbf{V}_l)$$

Molecular dynamics and DPD

- Relationship between MD and DPD
  - Ensemble averaging of material derivatives
    - Assuming MD particles interact via (hard) Lennard-Jones potential, following momentum flux equation applies:

$$\rho\langle\mathbf{\Pi}_i\rangle = m\rho\mathbf{v}\mathbf{v} + \mathbf{I}\rho - \mu(\nabla\mathbf{v} + (\nabla\mathbf{v})^T)$$

    - Using linear velocity gradients we (eventually) get:

$$\dot{\mathbf{P}}_k = \sum_i \tfrac{1}{2}\langle\dot{M}_{kl}\rangle(\mathbf{V}_k + \mathbf{V}_l) - \sum_l l_{kl}\left(\tfrac{1}{2}(p_k - p_l)\cdot\hat{\mathbf{r}}_{kl} + \frac{\mu}{r_{kl}}[\mathbf{V}_{kl} + (\mathbf{V}_{kl}\cdot\hat{\mathbf{r}}_{kl})\hat{\mathbf{r}}_{kl}]\right) + \sum_i \tilde{\mathbf{F}}_{kl}$$

Similar to: DPD conservative and dissipative forces

    - Momentum conservation, Galilean invariance

Molecular dynamics and DPD

- Relationship between MD and DPD
  - Ensemble averaging of material derivatives
    - Energy (carried out in similar way to momentum):

$$\dot{E}_k^T = -\sum_l l_{kl}\lambda\frac{(T_k - T_l)}{r_{kl}} - \sum_l l_{kl}\left(\frac{1}{2}(p_k + p_l)\cdot\hat{\mathbf{r}}_{kl} + \frac{\mu}{r_{kl}}[\mathbf{V}_{kl} + (\mathbf{V}_{kl}\cdot\hat{\mathbf{r}}_{kl})\hat{\mathbf{r}}_{kl}]\right)\cdot\frac{\mathbf{V}_{kl}}{2}$$
$$+\sum_l \frac{1}{2}\langle\dot{M}_{kl}\rangle\left(\frac{\mathbf{V}_{kl}}{2}\right)^2 + \frac{l_{kl}}{4r_{kl}}L_{kl}\hat{\mathbf{i}}_{kl}\cdot\mathbf{V}_{kl}\left(\frac{E_k}{V_k} + \frac{E_l}{V_l}\right) - \sum_i \tilde{\mathbf{F}}_{kl}\cdot\frac{\mathbf{V}_{kl}}{2} + \tilde{q}_{kl}$$

    - Potential energy term similar to DPD's
    - Terms similar to dissipative and random forces: DPD thermostat!

Molecular dynamics and DPD

- Relationship between MD and DPD
  - DPD can be derived from MD via coarse-graining and ensemble averaging
  - Hydrodynamics emergent from both MD and DPD
    - Thermostats often used in MD can break local momentum conservation (although global momentum is often correct)
    - DPD thermostat conserves both local and global momenta ‘automatically’
  - DPD parameters can be derived from MD simulations

DPD miscellany

- Can couple barostats to DPD thermostat to give constant pressure, surface area/tension ensembles[1]
  - Applied during force integration (Velocity Verlet) stages
  - Similar approach to classical MD
- Constant-energy DPD (DPD-E)[2] also available:
  - Can associate temperature for each particle
  - Integrate internal energy as well as velocity
  - Possible to couple with barostat to give constant enthalpy ensemble[3]

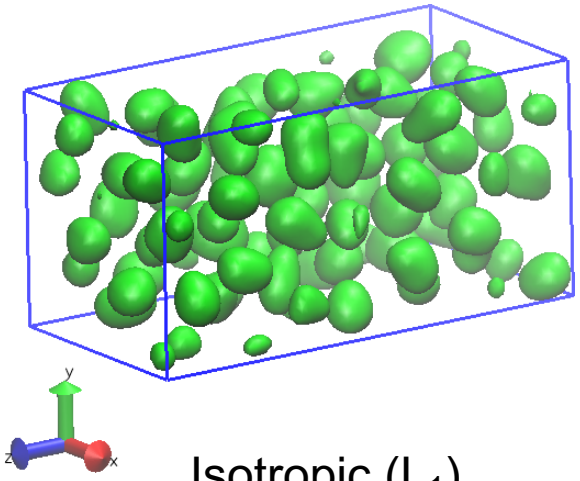
[1] Jakobsen, *J Chem Phys* **122**, 124901 (2005)
[2] Pastewka *et al.*, Phys Rev E 73, 037701 (2006)
[3] Lísal et al., J Chem Phys 135, 204105 (2011)

DPD Practical Exercises 2 and 3

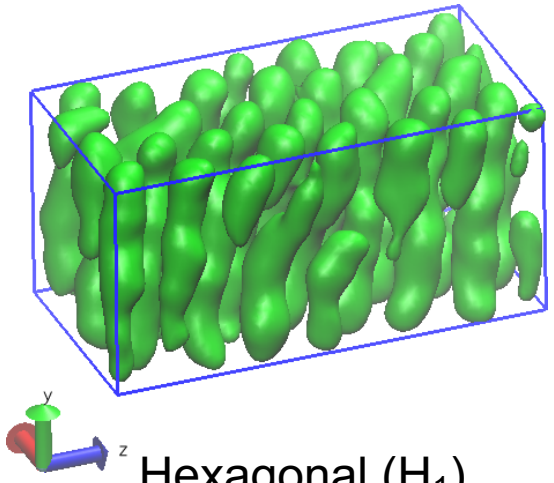
- Due to time, suggest that only one of these needs to be completed today: you can each choose which one
  - Course materials available after today, so can complete the other exercise later
- Exercise 2: Mesophase behaviour
  - Using DPD to find mesophase structures of two-bead amphiphilic molecules (dimers)
- Exercise 3: Transport properties
  - Finding viscosity of DPD fluid using Lees-Edwards boundary conditions (and trying out another pairwise thermostat)
- Start from drfaustroll.gitlab.io/cs2019 (Day 5: Dissipative Particle Dynamics)

DPD Practical Exercise 2

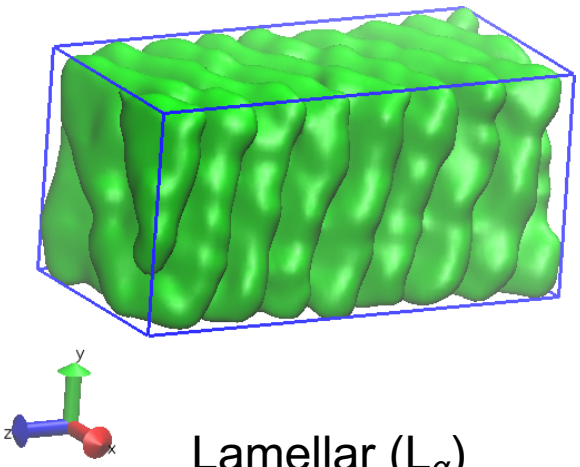
- Using DPD to find structures of two-bead amphiphilic molecules (dimers) at various concentrations
- Run simulations at different dimer concentrations and work out mesophasic structures formed
  - Visualise shapes, use order parameters
- Try and find boundaries between mesophases



Isotropic (L_1)

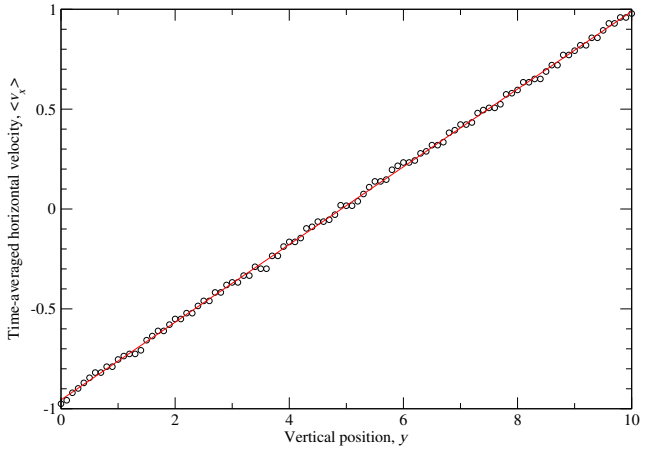


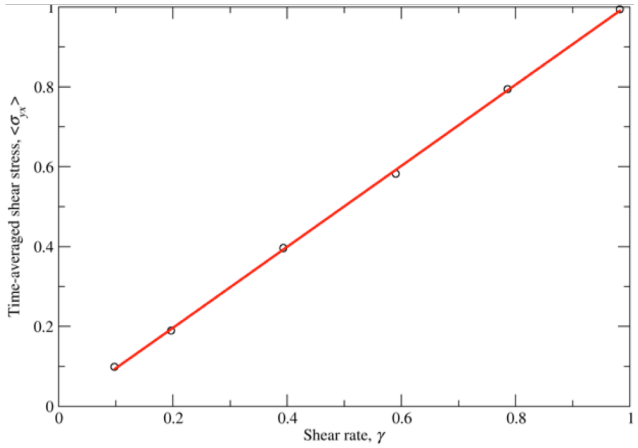
Hexagonal (H_1)



Lamellar (L_α)

DPD Practical Exercise 3





- Finding viscosity of particle-based fluid with pairwise thermostats (DPD, Stoyanov-Groot)
  - Apply linear shear to box to give constant shear rate (velocity gradient)
  - Measure shear stresses
  - Plot stress vs. shear rate: viscosity = gradient
- Try and find relationships between viscosity and thermostat parameter (γ or Γ)